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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION (UPDATED)

CLASS - 11 (PCM)
Question Paper Code : UN494

KEY

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SOLUTIONS

MATHEMATICS

01. (C) $2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha - 2 = 0$

$\Rightarrow$ $2\sin^2\alpha(\sin\alpha - 1) - 5\sin\alpha(\sin\alpha - 1) + 2(\sin\alpha - 1) = 0$

$\Rightarrow$ $(\sin\alpha - 1)(2\sin^2\alpha - 5\sin a + 2) = 0$

$\sin\alpha - 1 = 0$ or $2\sin^2\alpha - 5\sin\alpha + 2 = 0$

$\sin\alpha = 1$ or $\sin\alpha = \dfrac{5 \pm \sqrt{25-16}}{4} = \dfrac{5\pm 3}{4}$

$\alpha = \dfrac{\pi}{2}$ or $\sin\alpha = \dfrac{1}{2}, 2$

Now, $\sin\alpha \neq 2$

for, $\sin\alpha = \dfrac{1}{2}$

$\alpha = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

There are three values of α between $[0, 2\pi]$

02. (B) Coefficients of the 5th, 6th and 7th terms in the given expansion are ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$.

These coefficients are in AP.

Thus, we have

$2\,{}^nC_5 = {}^nC_4 + {}^nC_6$

On dividing both sides by we get

$$2 = \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5}$$

$$\Rightarrow 2 = \frac{5}{n-4} + \frac{n-5}{6}$$

$\Rightarrow$ 12n – 48 = 30 + n^2 – 4n – 5n + 20

$\Rightarrow$ n^2 – 21n + 98 = 0

$\Rightarrow$ (n – 14)(n – 7) = 0

$\Rightarrow$ n = 7, 14

03. (B) Product of two even number is always even and product of two odd numbers is always odd.

$\therefore$ Number of required subsets

= Total number of subsets – Total number of subsets

Having only odd numbers

= $2^{100} - 2^{50} = 2^{50}(2^{50} - 1)$

04. (D) $(x+iy)^{\frac{1}{3}} = a + ib$

Cubing on both the sides, we get

$x + iy = (a + ib)^3$

$\Rightarrow x + iy = a^3 + (ib)^3 + 3a^2bi + 3a(bi)^2$

$\Rightarrow x + iy = a^3 + i3b^3 + 3a^2bi + 3i^2ab^2$

$\Rightarrow x + iy = a^3 - ib^3 + 3a^2bi - 3ab^2$

($\because$ i2 = –1, i^3 = –i)

Þ $x + iy = a^3 - 3ab^2 + i(-b^3 + 3a^2b)$

$\therefore x = a^3 - 3ab^2$ and $y = 3a^2b - b^3$

or, $\frac{x}{a} = a^2 - 3b^2$ and $\frac{y}{b} = 3a^2 - b^2$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = a^2 - 3b^2 + 3a^2 - b^2$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = 4a^2 - 4b^2$$

05. (D) The given equation is

$\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta, \alpha,$ $\beta \in [0, \pi]$. Then, by A.M., G.M. inequality, A.M. $\geq$ G.M.

$$\frac{\sin^4\alpha + 4\cos^4\beta + 1 + 1}{4} \geq (\sin^4\alpha \cdot 4\cos^4\beta \cdot 1 \cdot 1)^{\frac{1}{4}}$$

$$\sin^4\alpha + 4\cos^4\beta + 1 + 1 \geq 4\sqrt{2}\sin\alpha \cdot |\cos\beta|$$

Inequality still holds when $\cos\beta < 0$ but L.H.S. is positive than $\cos\beta > 0$, then

L.H.S. = R.H.S

$\therefore$ $\sin^4\alpha = 1$ and $\cos^4\beta = \frac{1}{4}$

$\Rightarrow$ $\alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{4}$

$\therefore$ $\cos(\alpha+\beta) - \cos(\alpha-\beta)$

$$= \cos\left(\frac{\pi}{2}+\beta\right) - \cos\left(\frac{\pi}{2}-\beta\right)$$

$$= -\sin\beta - \sin\beta = -2\sin\frac{\pi}{4} = -\sqrt{2}$$

06. (A) $K = \frac{x^2 - x + 1}{x^2 + x + 1}$

$\Rightarrow kx^2 + kx + K = x^2 - x + 1$

$\Rightarrow (k-1)x^2 + (k+1)x + k - 1 = 0$

For real values of x, the discriminate of $(k-1)x^2 + (k+1)x + k - 1 = 0$ should be greater than or equal to zero.

$\therefore$ If k $\neq$ 1

$(k+1)^2 - 4(k-1)(k-1) > 0$

$\Rightarrow (k+1)^2 - \{2(k-1)\}^2 > 0$

$\Rightarrow (3k-1)(k-3) < 0$

$$\Rightarrow \frac{1}{3} < K < 3$$

And if k = 1, then

x = 0, which is real ...... (ii)

So, from (i) and (ii) we get

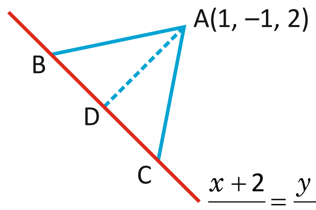
$$K \in \left[\frac{1}{3}, 3\right]$$

07. (D) Let a point D on BC = $(3\lambda - 2, 1, 4\lambda)$

$$\overrightarrow{AD} = (3\lambda - 3)\hat{i} + 2\hat{j} + (4\lambda - 2)\hat{k}$$

$$\because \overrightarrow{AD} \perp \overrightarrow{BC}, \therefore \overrightarrow{AD}, \overrightarrow{BC} = 0$$

$$\Rightarrow (3\lambda - 3) + 3 + 2(0) + (4\lambda - 2)4 = 0 \Rightarrow \lambda = \frac{17}{25}$$



$$\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$$

Hence, $D = \left(\frac{1}{25}, 1, \frac{68}{25}\right)$

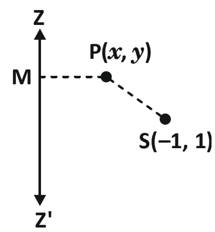
$$\left|\overrightarrow{AD}\right| = \sqrt{\left(\frac{1}{25} - 1\right)^2 (2)^2 + \left(\frac{68}{25} - 2\right)^2}$$

$$= \sqrt{\frac{(24)^2 + 4(25)^2 + (18)^2}{25}} = \sqrt{\frac{3400}{24}} = \frac{2\sqrt{34}}{5}$$

Area of triangle $= \frac{1}{2} \times \left|\overrightarrow{BC}\right| \times \left|\overrightarrow{AD}\right|$

$$= \frac{1}{2} \times 5 \times \frac{2\sqrt{34}}{5} = \sqrt{34} \quad [\because BC = 5]$$

08. (B) $7x^2 + 2xy + 7y^2 + 10x - 10y + 7 = 0$



Let $P(x, y)$ be any point on the ellipse whose focus and eccentricity are $S(-1, 1)$ and $e = \frac{1}{2}$

Then SP = e × PM

$\Rightarrow$ SP = $\frac{1}{2}$ × PM

$\Rightarrow$ 2SP = PM

$\Rightarrow 4(SP)^2 = PM^2$

$$\Rightarrow 4\left[(x+1)^2 + (y-1)^2\right] = \left(\frac{x - y + 3}{\sqrt{1^2 + (-1)^2}}\right)^2$$

$\Rightarrow 4[x^2 + 1 + 2x + y^2 + 1 - 2y]$

$$= \frac{x^2 + y^2 + 9 - 2xy - 6y + 6x}{2}$$

$\Rightarrow 8x^2 + 8 + 16x + 8y^2 + 8 - 16y$

$\Rightarrow x^2 + y62 + 9 - 2xy - 6y + 6x$

$\therefore 7x^2 + 7y^2 + 2xy - 10y + 10x + 7 = 0$

This is the required equation of the ellipse.

09. (A) The thousands place can only be filled with 2, 3 or 4, since the number is greater than 2000.

For the remaining 3 places, we have pick out digits such that the resultant number is divisible by 3.

It the sum of digits of the number is divisible by 3, then the number itself is divisible by 3.

Case 1: If we take 2 at thousands place.

The remaining digits can be filled as:

0, 1 and 3 as 2 + 1 + 0 + 3 = 6 is divisible by 3.

0, 3 and 4 as 2+ 3 + 0 + 4 = 9 is divisible by 3.

In both the above combinations the remaining three digits can be arranged in 3! ways.

$\therefore$ Total number of numbers in this case = 2 × 3! = 12.

Case 2: If we take 3 at thousands place. The remaining digits can be filled as:

0, 1 and 2 as 3 + 1 + 0 + 2 = 6 is divisible by 3.

0, 2 and 4 as 3 + 2 + 0 + 4 = 9 is divisible by 3.

In both the above combinations, the remaining three digits can be arranged in 3! ways. Total number of numbers in this case = 2 × 3! = 12.

Case 3: If we take 4 at thousands place.

The remaining digits can be filled as:

0, 2 and 3 as 4 + 2 + 0 + 3 = 9 is divisible by 3.

In the above combination, the remaining three digits can be arranged in 3! ways.

$\therefore$ Total number of numbers in this case = 3! = 6.

$\therefore$ Total number of numbers between 2000 and 5000 divisible by 3 are 12 + 12 + 6 = 30.

10. (B) Let Δ ABC be in the first quadrant

Slope of line AB = $-\frac{1}{2}$

Slope of line AC = 2

Length of AB = $\sqrt{5}$

It is given that are (Δ ABC) = $5\sqrt{5}$

$\therefore$ $\frac{1}{2}AB \cdot AC = 5\sqrt{5} \Rightarrow AC = 10$

$\therefore$ Coordinate of vertex C = (1 + 10cosθ, 2 + 10 sinθ)

$\because$ $\tan\theta = 2 \Rightarrow \cos\theta = \frac{1}{\sqrt{5}}, \sin\theta = \frac{2}{\sqrt{5}}$

$\therefore$ Coordinate of C = (1 + 2$\sqrt{5}$, 2 + 4$\sqrt{5}$)

$\therefore$ Abscissa of vertex C is 1 + 2$\sqrt{5}$

11. (D) Suppose : A(5, –4, 2) ; B(4, –3, 1)

C(7, 6, 4) ; D(8, –7, 5)

$$AB = \sqrt{(4-5)^2 + (-3+4)^2 + (1-2)^2}$$

$$= \sqrt{(-1)^2 + (1)^2 + (-1)^2}$$

$$= \sqrt{1+1+1} = \sqrt{3}$$

$$BC = \sqrt{(7-4)^2 + (6+3)^2 + (4-1)^2}$$

$$= \sqrt{(3)^2 + (9)^2 + (3)^2}$$

$$= \sqrt{9+91+9} = \sqrt{99} = 3\sqrt{11}$$

$$CD = \sqrt{(8-7)^2 + (-7-6)^2 + (5-4)^2}$$

$$= \sqrt{(1)^2 + (-13)^2 + (1)^2}$$

$$= \sqrt{1+169+1} = \sqrt{171}$$

$$DA = \sqrt{(8-5)^2 + (-7+4)^2 + (5-2)^2}$$

$$= \sqrt{(3)^2 + (-3)^2 + (3)^2}$$

$$= \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$$

We see that none of the sides are equal.

12. (A) let f(x) = $\log\left\{e^x\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right\}$

$$= \log e^x + \log\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}$$

$$= x\log e + \frac{3}{4}\log\left(\frac{x-2}{x+2}\right)$$

$$= x + \frac{3}{4}\log\left(\frac{x-2}{x+2}\right) \quad [\because \log e = 1]$$

Now, $\frac{d}{dx}(f(x)) = 1 + \frac{3}{4}\frac{d}{dx}\log\left(\frac{x-2}{x+2}\right)$

$$= 1 + \frac{3}{4}\left[\frac{x+2}{x-2}\left\{\frac{(x+2).1-(x-2).1}{(x+2)^2}\right\}\right]$$

$$= 1 + \frac{3}{4} \times \frac{4}{x^2-4}$$

$$= 1 + \frac{3}{x^2-4}$$

$$\Rightarrow \frac{d}{dx}(f(x))\Big|_{\text{at } x=3} = 1 + \frac{3}{5} = \frac{8}{5}$$

13. (B) $f(x)=1+x+\frac{x^2}{2}+...+\frac{x^{100}}{100}$

Differentiate both the sides with respect to x, we get

$$f'(x)=\frac{d}{dx}\left(1+x+\frac{x^2}{2}+...+\frac{x^{100}}{100}\right)$$

$$=\frac{d}{dx}(1)+\frac{d}{dx}(x)+\frac{d}{dx}\left(\frac{x^2}{2}\right)+...+\frac{d}{dx}\left(\frac{x^{100}}{100}\right)$$

$$=\frac{d}{dx}(1)+\frac{d}{dx}(x)+\frac{1}{2}\frac{d}{dx}(x^2)+...+\frac{1}{100}\frac{d}{dx}(x^{100})$$

$$=0+1+\frac{1}{2}\times 2x+...+\frac{1}{100}\times 100x^{99}$$

$= 1 + x + x^2 + . . . + x^{99}$

Putting x = 1, we get

$f'(x)$ = 1 + 1 + 1 + . . . + 1 (100 terms)

= 100

14. (D) We have

$$^{37}C_4+\sum_{r=1}^{5}{}^{(42-r)}C_r$$

$$={}^{37}C_4+{}^{41}C_1+{}^{40}C_2+{}^{39}C_3+{}^{38}C_3+{}^{37}C_3$$

$$={}^{37}C_3+{}^{37}C_4+{}^{38}C_3+{}^{39}C_3+{}^{40}C_3+{}^{41}C_3$$

$$={}^{38}C_4+{}^{38}C_3+{}^{39}C_3+{}^{40}C_3+{}^{41}C_3$$

$[\because {}^{n}C_{r-1}+{}^{n}C_r={}^{n+1}C_r]$

$$={}^{39}C_4+{}^{39}C_3+{}^{40}C_3+{}^{41}C_3$$

$$={}^{40}C_4+{}^{40}C_3+{}^{41}C_3$$

$$={}^{41}C_4+{}^{41}C_3$$

$$={}^{42}C_4$$

15. (D) $|x + 3| \geq 10$

$\Rightarrow x + 3 \geq 10$ or $x + 3$; $\leq - 10$

$\Rightarrow x \geq 10 - 3$ or $x \leq - 10 - 3$

$\Rightarrow x \geq 7$ or $x \leq -13$

$\Rightarrow x \in (-\infty, -13] \cup [7, \infty)$

16. (A) The y-coordinate of foci is zero

$\therefore$ Major axis is on X-axis ae = 4

Let, equation of ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$[\because b^2 = a^2(1 - e^2) = a^2 - 16]$

$$\Rightarrow \frac{32}{a^2}+\frac{24}{a^2-16}=1$$

$\Rightarrow 32a^2 - 512 + 24a^2 = a^2(a^2 - 16)$

$\Rightarrow 56a^2 - 512 = a^4 - 16a^2$

$\Rightarrow a^4 - 72a^2 + 512 = 0$

$\Rightarrow a^2 - 64a^2 - 8a^2 + 512 = 0$

$\Rightarrow a^2(a^2 - 64) - 8(a^2 - 64) = 0$

$\Rightarrow (a^2 - 8)(a^2 - 64) = 0$

$\Rightarrow a^2 = 64 \Rightarrow a = 8$

$(\because a^2 = 8$ is not possible)

$\because$ ae = 4 $\Rightarrow$ 8 × e = 4

$\Rightarrow e = \frac{1}{2}$

17. (C) Given, ABCD is a parallelogram with vertices

A(4, 4, –1), B(5, 6, – 1), C(6, 5, 1) and D(x, y, z).

We know that diagonals of parallelogram ABCD bisects each other.

$\therefore$ Mid-point of AC = Mid-Point of BD

$$\Rightarrow\left(\frac{4+6}{2},\frac{4+5}{2},\frac{-1+1}{2}\right)=\left(\frac{x+5}{2},\frac{y+6}{2},\frac{z-1}{2}\right)$$

$$\Rightarrow\left(\frac{10}{2},\frac{9}{2},0\right)=\left(\frac{x+5}{2},\frac{y+6}{2},\frac{z-1}{2}\right)$$

On comparing both sides, we get

$$\frac{x+5}{2}=\frac{10}{2},\frac{y+6}{2}=\frac{9}{2}\text{and}\frac{z-1}{2}=0$$

$\Rightarrow x + 5 = 10, y + 6 = 9$ and $z - 1 = 0$

$\Rightarrow x = 10 - 5, y = 9 - 6$ and $z = 1$

$\Rightarrow x = 5, y = 3$ and $z = 1$

Thus, D(x, y, z) = D (5, 3, 1)

18. (B) We need at least three points to draw a circle that passes through them.

Now, number of circles formed out of 11 points by taking three points at a time = $^{11}C_3$ = 165

Number of circles formed out of 5 points by taking three points at a time = $^{5}C_3$ = 10

It is given that 5 points lie on one circle.

Required number of circles

= 165 – 10 + 1 = 156

19. (B) We have,

$$\overline{Z}^{\frac{1}{3}} = a + ib$$

$$\Rightarrow \overline{Z} = (a+ib)^3$$

$$\Rightarrow x - iy = (a+ib)^3 \quad [\because \bar{z} = x - iy]$$

$$\Rightarrow x - iy = a^3 + i^3b^3 + 3a^2(ib) + 3a(i^2b^2)$$

$$\Rightarrow x - iy = a^3 - ib^3 + 3a^2bi - 3ab^2$$

$$\Rightarrow x - iy = (a^3 - 3ab^2) + i(3a^2b - b^3)$$

$$\Rightarrow x = a^3 - 3ab^2 \text{ and } y = -3a^2b + b^3$$

$$\Rightarrow \frac{x}{a} = a^2 - 3b^2 \text{and} \frac{y}{b} = -3a^2 + b^2$$

Now, $\frac{x}{a} + \frac{y}{b} = a^2 - 3b^2 - 3a^2 + b^2$

= – 2a² – 2b²

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = -2(a^2 + b^2)$$

$$\therefore \frac{1}{a^2+b^2}\left(\frac{x}{a} + \frac{y}{b}\right) = -2$$

20. (C) If the numbers of nails and nuts are 6 and 10, respectively, then the numbers of rusted nails and rusted nuts are 3 and 5, respectively.

Total number of items = 6 + 10 = 16

Total number of rusted items = 3 + 5 = 8

Total number of ways of drawing one item = $^{16}C_1$

Let R and N be the events where both the items drawn are rusted items and nails, respectively.

R and N are not mutually exclusive events, because there are 3 rusted nails.

P(either a rusted item or a nail)

= P(R ∪ N)

= P(R) + P(N) – P(R ∩ N)

$$\frac{^6C_1}{^{16}C_1} + \frac{^8C_1}{^{16}C_1} - \frac{^3C_1}{^{16}C_1}$$

$$= \frac{6}{16} + \frac{8}{16} - \frac{3}{16} = \frac{11}{16}$$

21. (B) $f(x) = \sqrt{2 - 2x - x^2}$

Since, $2 - 2x - x^2 \geq 0$

$x^2 + 2x - 2 \leq 0$

$\Rightarrow x^2 - 2x - 2 + 1 - 1 \leq 0$

$$\Rightarrow (x-1)^2 \left(\sqrt{3}\right)^2 \leq 0$$

$$\Rightarrow \left[x - \left(1 - \sqrt{3}\right)\right]\left[x - \left(1 + \sqrt{3}\right)\right] \leq 0$$

$$\Rightarrow \left(-1 - \sqrt{3}\right) \leq x \leq \left(-1 + \sqrt{3}\right)$$

Thus, domain (f) = $\left[-1-\sqrt{3}, -1+\sqrt{3}\right]$

22. (A) $f(x) = \sin[\pi^2]x + \sin[-\pi^2]x$

$\Rightarrow f(x) = \sin[9.8]x + \sin[-9.8]x$

$\Rightarrow f(x) = \sin 9x - \sin 10x$

$$\Rightarrow f\left(\frac{\pi}{2}\right) = \sin 9 \times \frac{\pi}{2} - \sin 10 \times \frac{\pi}{2}$$

$$\Rightarrow f\left(\frac{\pi}{2}\right) = 1 - 0 = 1$$

23. (C) Given

$\Rightarrow \cos(5x - 2x)\tan 5x = \sin(5x + 2x)$

$$\Rightarrow \tan 5x = \frac{\sin(5x + 2x)}{\cos(5x - 2x)}$$

$$\Rightarrow \tan 5x = \frac{\sin 5x \cos 2x + \cos 5x \sin 2x}{\cos 5x \cos 2x + \sin 5x \cos 2x}$$

$$\Rightarrow \frac{\sin 5x}{\cos 5x} = \frac{\sin 5x \cos 2x + \cos 5x \sin 2x}{\cos 5x \cos 2x + \sin 5x \cos 2x}$$

$\Rightarrow$ sin5 $x$cos5x + sin^{2}5$x$sin2x

= sin5$x$cos5$x$cos2x + cos^{2}5$x$sin2x

$\Rightarrow$ sin^{2}5$x$sin2x = cos^{2}5$x$sin2x

$\Rightarrow$ (sin^{2}5$x$cos^{2}5x)sin2x = 0

$\Rightarrow$ (sin5x – cos5x)(sin5x – cos5x)sin2x = 0

$\Rightarrow$ sin5x – cos5x = 0, sin5x + cos5x = 0 or sin2x = 0

$$\Rightarrow \frac{\sin 5x}{\cos 5x}=1, \frac{\sin 5x}{\cos 5x}=-1 \text{ or } \sin 2x=0$$

Now,

$$\Rightarrow \tan 5x=\tan\frac{\pi}{4}$$

$$\Rightarrow 5x=n\pi+\frac{\pi}{4}, n\in Z$$

$$\Rightarrow x=\frac{n\pi}{5}+\frac{\pi}{20}, n\in Z$$

F or n = 0, 1 and 2, the values of x are $\frac{\pi}{20}, \frac{\pi}{4}$ and $\frac{9\pi}{20}$ respectively.

Or

tan5x = 1

$$\Rightarrow \tan 5x=\tan\frac{3\pi}{4}$$

$$\Rightarrow 5\pi=n\pi+\frac{3\pi}{4}, n\in Z$$

$$\Rightarrow xx=\frac{n\pi}{5}+\frac{3\pi}{20}, n\in Z$$

For n = 0 and 1, the value of x are $\frac{3\pi}{20}$ and $\frac{7\pi}{20}$ respectively.

And,

sin2x = sin0

$\Rightarrow$ sin2x = sin0

$\Rightarrow$ 2x = nπ, n $\in$ Z

$$\Rightarrow x=\frac{n\pi}{2}, n\in Z$$

For n = 0, the value of x is 0

Also for the odd multiple of $\frac{\pi}{2}$ tan$x$ is not defined.

Hence, there are six solutions.

24. (A) Let the reflection point be A(h, k)

Now, the mid point of line joining (h, k) and (4, –13) will lie on the line 5x + y + 6 = 0

$$\therefore 5\left(\frac{h+4}{2}\right)+\frac{k-13}{2}+6=0$$

$\Rightarrow$ 5h + 20 + k – 13 + 12 = 0

$\Rightarrow$ 5h + k + 19 = 0 .... (1)

Now, the slope of the line joining (h, k) and (4, –13) are perpendicular to the line 5x + y + 6 = 0

slope of the line = –5

slope of line joining by points (h, k) and (4, –13)

$$\frac{k+13}{h-4}$$

$$\therefore \frac{k+13}{h-4}(-5)=-1$$

$\Rightarrow$ 5k – h + 60 = 0 .... (2)

Solving (1) and (2) we get

h = –1 and k = –14

25. (D) The truth table of both the statements is

p	q	~p	~q	$q \vee p$	$p \leftrightarrow \sim q$	(S_1)	$\sim p \leftrightarrow q$	(S_2)
T	T	F	F	T	F	F	F	F
T	F	F	T	T	T	T	T	T
F	T	T	F	T	T	T	T	F
F	F	T	T	F	F	T	F	F

$\therefore$ S_1 is not tautology and

S_2 is not fallacy

Hence, both the statements (S_1) and (S_2) are not correct.

PHYSICS

26. (C) Average speed is never less than average velocity. Average velocity of a particle moving once around a circle can be zero but instantaneous velocity is never zero in the interval.

Average velocity of a particle moving on a straight line is never zero.

When a particle is in vertical motion, then at the highest point, the instantaneous velocity of the particle is zero but the acceleration is not zero.

27. (C) Impulse delivered to the ball = F × t

= Area enclosed by the force-time graph with the x-axis.

$= \frac{1}{2} \times 1.5 \times 1750$

= 1312.5 N s

The average force exerted on the ball

$= \frac{\text{Impulse}}{\text{Time}} = \frac{1312.5}{1.5} = 875 \text{ N}$

28. (C) The graph between applied force and extension will be straight line because in elastic range,

Applied force $\propto$ extension,

But the graph between extension and stored elastic energy will be parabolic in nature.

As, $U = \frac{1}{2} kx^2$ or $U \propto x^2$

29. (B) $mgh = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$

$= \frac{1}{2} I\omega^2 + \frac{1}{2} mr^2\omega^2 = \frac{\omega^2}{2} [I + mr^2]$

$$\omega = \left[\frac{2\,mgh}{I + mr^2}\right]^{1/2}$$

30. (B) Mean diameter

$$\frac{0.39 + 0.38 + 0.39 + 0.41 + 0.38 + 0.37 + 0.40 + 0.39}{8}$$

$\bar{d} = 0.38875$ mm

= 0.39 mm (rounded off to two significant figures)

Absolute error in the first reading

= 0.39 – 0.39 = 0.00 mm

Similarly finding the absolute error in the other seven readings and taking the mean ;

Mean absolute error

$$\overline{\Delta d} = \frac{0.00 + 0.01 + 0.00 + 0.02 + 0.01 + 0.02 + 0.01 + 0.00}{8}$$

= 0.00875 = 0.01 mm

Relative error = $\frac{\overline{\Delta d}}{\bar{d}} = \frac{0.01}{0.39} = 0.0256$

31. (B) $\vec{A} + \vec{B}$ is the resultant of $\vec{A}$ and $\vec{B}$. Let $|\vec{A}| = |\vec{B}| = x = |\vec{A} + \vec{B}|$, $x^2 = x^2 + x^2 + 2x^2 \cos\theta$, $\cos\theta = -1/2$, $\theta = 120°$

When $\vec{R} = \vec{A} - \vec{B} = x$, $x^2 = x^2 + x^2 - 2x^2 \cos\theta$, $\cos\theta = +1/2$, $\theta = 60°$

32. (D) The acceleration due to gravity at a depth d inside the earth is

$$g' = g\left(1 - \frac{d}{R}\right) = g\left(\frac{R - d}{R}\right) = g\frac{r}{R}$$

Where, R – d = r = distance of a place from the centre of earth. Therefore 'g' α r.

33. (C) We can assume that the process is isothermal because the temperature of the surrounding remains constant

So, by applying Boyle's law

$P_s V_s = P_d V_d$ .... (I)

Where s = surface of water

d = depth

Given that Depth of lake = h

Volume $V_s = 3V_d$

P_s = 75 cm Hg

$P_d = 75 + \frac{h}{10}$

Put the value of equation (1)

$75 \times 3 = 75 + \frac{h}{10}$

$225 - 75 = \frac{h}{10}$

h = 1500 cm

h = 15 m

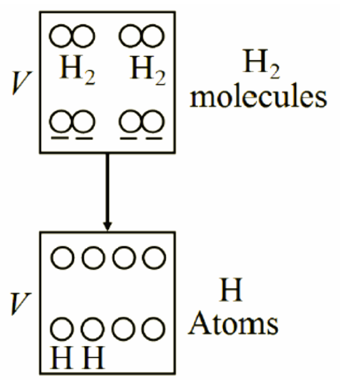
Hence, the depth of lake is 15 m

34. (C) The work done by a variable force is defined as $W = \int \vec{F}.\vec{ds}$

- It may or may not depend on the path followed.
- It is always dependent on the initial and final positions.

35. (D) The situation is shown in the diagram given below. H_2 gas is contained in a box is heated and gets converted to a gas of hydrogen atoms. Then the number of moles would become twice.

According to gas equation, PV = nRT



P = Pressure of gas, n = Number of moles

R = Gas constant, T = Temperature PV = nRT

As volume (V) of the container is constant.

Hence, when temperature (T) becomes 10 times, (from 300K to 3000K) pressure (P) also becomes 10 times, as $P \propto T$.

Pressure is due to the bombardment of particles and as gases break, the number of moles becomes twice of initial, so $n_2 = 2n_1$

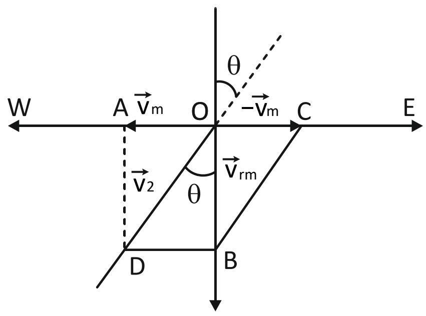
So, $P \propto nT$

$\Rightarrow \frac{P_2}{P_1} = \frac{n_2T_2}{n_1T_1} = \frac{(2n_1)(3000)}{n_1(300)} = 20$

$\Rightarrow P_2 = 20\ P_1$

Hence, final pressure of the gas would be 20 times the pressure initially.

36. (C) Observe the figure given below.



$\vec{v}_m$ = 6 km/h due west = $(\overrightarrow{OA})$

$\vec{v}_{rm}$ = 6 km/h vertically downwards

= $(\overrightarrow{OB})$

Let $\vec{v}_r$ = true velocity of rain. It must be represented by $(\overrightarrow{OD})$

$\therefore \vec{v}_{rm} = \vec{v}_r + (-\vec{v}_m)$

or $\vec{v}_r = \vec{v}_{rm} - (-\vec{v}_m) = (\vec{v}_{rm} + \vec{v}_m)$

$= (\overrightarrow{OB}) + (\overrightarrow{OA}) = (\overrightarrow{OD})$

where OD = $\sqrt{6^2 + 6^2}$

$= 6\sqrt{2}$ km/h

If $\angle BOD = \theta$, then

$\tan\theta = \frac{BD}{OB} = \frac{6}{6} = 1 = \tan 45°$

or $\theta = 45°$

$\therefore$ velocity of rain is $6\sqrt{2}$ km/h at angle 45° with the vertical towards east.

37. (B) Let us assume that $T_1 > T_2, T_3$ and $T_1 > T > T_2, T_3$

Now heat loss by M_1 = Heat gained by M_2 and M_3

$M_1S(T_1 - T) = M_2S(T - T_1) + M_3S(T - T_3)$

$\Rightarrow$ $M_1T_1 + M_2T_2 + M_3T_3 = (M_1 + M_2 + M_3)T$

$$\Rightarrow \quad T = \frac{M_1T_1 + M_2T_2 + M_3T_3}{M_1 + M_2 + M_3}$$

38. (A) Escape velocity from the surface of Mars.

$$v = \sqrt{\frac{2GM_m}{R_m}}$$

Mass of Mars = $M_m = 6.42 \times 10^{23}$ kg

Radius of Mars = $R_m = 3.375 \times 10^6$ m

$$v = \sqrt{\frac{2GM_m}{R_m}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6.42 \times 10^{23}}{3.375 \times 10^6}}$$

$= 5.037 \times 10^3$ m/s

39. (A) $T = P^a D^b S^c$

So, $[T] = [M^0 L^0 T^1] = [ML^{-1}T^{-2}][ML^{-3}]^b[MT^{-2}]^c$

$= [M^{a+b+c} L^{-a-3b} T^{-2a-2c}]$

Applying principle of homogeneity

$a + b + c = 0$; $-a - 3b = 0$; $-2a - 2c = 1$

On solving, we get $a = -\frac{3}{2}$, $b = \frac{1}{2}$, $c = 1$

40. (B) Friction is absent. Therefore, mechanical energy of the system will remain conserved. From constraint relations we see that speed of both the blocks will be same. Suppose it is v. Here gravitational potential energy of 2 kg block is decreasing while gravitational potential energy of 1 kg block is increasing. Similarly, kinetic energy of both the blocks is also increasing.

Decrease in gravitational potential energy of 2 kg block = increase in gravitational potential energy of 1 kg block + increase in kinetic energy of 1 kg block + increase in kinetic energy of 2 kg block.

$$m_Q gh = m_P gh + \frac{1}{2}m_P v^2 + \frac{1}{2}m_Q v^2$$

or $(2)(10)(1) = (1)(10)(1) + \frac{1}{2}(1)v^2 + \frac{1}{2}(2)v^2$

or $20 = 10 + 0.5\,v^2 + v^2$

or $1.5\,v^2 = 10$

$\therefore$ $v^2 = 6.67\ m^2/s^2$

or $v = 2.58$ m/s

CHEMISTRY

41. (C) When an electron is added to O^- anion, there is strong electrostatic repulsion between the two negative charges. Due to this, the second electron gain enthalpy of oxygen is positive.

Thus, process of formation of O^{2-} in gas phase is unfavourable even though O^{2-} is isoelectronic with neon. It is due to the fact that electron repulsion outweighs the stability gained by achieving noble gas configuration.

42. (D) Higher the critical temperature, more easily is the gas liquefied. Hence, the order of liquefaction of given gases with the gas liquefying first will be O_2, N_2, H_2, He.

43. (A)

	$CH_3COOH + H_2O \rightleftharpoons$	H_3O^+	$+ CH_3OOH^-$
Initial conc.	0.01	0	0
At equilibrium	$0.01 - x$	x	x

$$K_a = \frac{[H_3O^+][CH_3COO^-]}{(CH_3COOH)} = \frac{x^2}{0.01 - x}$$

Since $x << 0.01$,

Therefore, $0.01 - x \approx 0.01$

$$\frac{x^2}{0.01} = 1.74 \times 10^{-5}$$

$x^2 = 1.74 \times 10^{-7}$ or $x = 4.2 \times 10^{-4}$

$pH = -\log(4.2 \times 10^{-4}) = 3.4$

44. (D) As we know that

Number of atoms = Mol. × N_A

Number of moles = $\frac{\text{Wt.}}{\text{Mol.wt.}}$

1. 4g He = $\frac{4}{4}$ – 1 mole

2. 46g Na = $\frac{46}{23}$ = 2 moles

3. 0.40g Ca = $\frac{0.40}{40}$ = 0.01 mole

4. 12g He = $\frac{12}{4}$ = 3 moles

Hence, 12 g of He contains the greatest number of atoms at it contains maximum number of moles.

45. (C) The second period elements of p-block starting from boron are restricted to a maximum covalency of four (using one 2s and three 2p orbitals).

46. (Delete)

47. (A) $Be(OH)_2$ being an amphoteric hydroxide reacts with both alkalies and acids as given below.

$Be(OH)_2 + 2NaOH \rightarrow Na_2BeO_2 + 2H_2O$

$Be(OH)_2 + 2HCl \rightarrow BeCl_2 + 2H_2O$

48. (A) H_2O_2 acts as an oxidising as well as reducing agent because oxidation number of oxygen in H_2O_2 is –1.

Rest of the given statements are correct.

49. (D) Boiling point of a liquid is directly proportional to atmospheric pressure. Shimla has the lowest atmospheric pressure. Hence, the liquid in Shimla will boil first.

50. (D) Highest O.N. of any transition element = (n – 1)d electrons +ns electrons. Therefore, larger the number of electrons in the 3d orbitals, higher is the maximum O.N.

(A) $3d^1 4s^2$ = 1 + 2 = 3

(B) $3d^3 4s^2$ = 3 + 2 = 5

(C) $3d^5 4s^1$ = 5 + 1 = 6

(D) $3d^5 4s^2$ = 5 + 2 = 7

Hence, the element with outer electronic configuration of $3d^5 4s^2$ exhibits largest oxidation number.

51. (B) According to the first law of thermodynamics,

$\Delta E = q + W = 500 + (-350) = +150$ cal

52. (C) sp-hybridisation involves the mixing of one s and one p-orbital resulting in the formation of two equivalent sp-hybrid orbitals. The suitable orbitals for sp-hybridisation are s and P_z, if the hybrid orbitals lie along the z-axis.

53. (B) Solubility of metal halides in water depends on lower lattice enthalpy and hydration enthalpy. In case of LiF, the lowest solubility in water is due to its very high lattice enthalpy.

54. (B) As per De Broglie equation

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1 \times 10^{3}}$$

$= 3.97 \times 10^{-9}$m = 0.40 nm

55. (C) When the electrophile attacks $CH_3 - CH = CH_2$, delocalisation of electrons can take place in two possible ways.

$CH_3 - CH = CH_2 + H^+ \rightarrow CH_3 - \overset{\oplus}{CH} - CH_3$ (2° carbocation)

$CH_3 - CH_2 - \overset{\oplus}{CH_2}$ (1° carbocation)

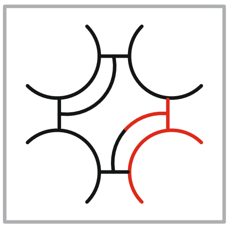
As 2° carbocation is more stable than 1° carbocation, the first addition is more feasible.

CRITICAL THINKING

56. (C) The ladder in picture R is the least stable or most likely to slip.

57. (A) Suspension of trade agreements between the two countries directly correlates with an economic impact.

58. (D)



59. (B) Given the rules, let's first list down the contacts.

1. B contacts E, contacted by C.
2. D contacted by A, E, F.
3. F contacts C, contacted by A.
4. D contacts C and E.

Now, to relay a message from A to B:

Starting with A, we see the robots A can directly contact or who can contact A. From the rules, A can contact D and F. B can only be contacted by C, so our goal is to find the shortest path from A to C.

Using the given rules:

$A \rightarrow D \rightarrow C$ (A contacts D and D contacts C)

OR

$A \rightarrow F \rightarrow C$ (A contacts F and F contacts C)

Both paths have two robots between A and B (either D and C or F and C).

Therefore, the minimum number of robots required between robot A and robot B to send a message is 2.

60. (B)